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THERMAL INTERACTION OF GEOTHERMAL BOREHOLES WITH GROUNDWATER FLOWS AT PECLET NUMBERS OF ORDER UNITY

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ABSTRACT

The presence of aquifers significantly affects the thermal response of geothermal boreholes. Consequently, theoretical models used in the design and sizing of geothermal heat exchangers must account for their presence. Existing models in the literature are effective for creeping groundwater flows, where heat convection close to the borehole is weak compared to heat conduction and the resulting Peclet number is small compared to unity. But strong groundwater flows, with Peclet numbers of order unity and beyond, are also present in real-world applications, such as in high-permeability soils or when energy piles are involved. In these cases, current models fail to correctly account for the flow field in the vicinity of the boreholes, leading to unsatisfactory results or the need of empirical tuning parameters. By using asymptotic expansion techniques, a new theoretical model for the thermal interaction of geothermal boreholes with strong groundwater flows has been developed. It is physically sound, mathematically rigorous, and free of tuning parameters. Comparisons with detailed numerical simulations demonstrate the models' promising performance and accuracy. Furthermore, the developed model is applicable to other heat transfer problems involving circular cylinders and potential or Darcy flows, thereby extending its relevance beyond the field of low-temperature geothermal energy.

KEY WORDS: HVAC systems, geothermal heat exchangers, geothermal boreholes, groundwater flows, asymptotic solution, Mathieu functions

1. INTRODUCTION

Heating and cooling of buildings represents almost 23% of mankind's global energy consumption [1]. With such a high share, decarbonization of heating, ventilation, and air conditioning (HVAC) systems emerges as one of the top priorities in the roadmaps of many countries and regions. HVAC systems that harness geothermal energy are among the favorite options for the sought energy transition. Geothermal HVAC systems incorporate an electricity-driven water-to-water heat pump connected to several vertical boreholes that collectively form the geothermal heat exchanger. As shown in Figure 1, each borehole includes multiple pipes forming coaxial or U-shaped probes. These probes facilitate the flow of a heat carrying liquid, allowing the exchange of heat with the surrounding ground. The space between pipes and ground is typically filled with impermeable grout to enhance the heat exchange and prevent the potential cross-contamination of aquifers.

The correct sizing of geothermal heat exchangers is crucial for optimizing both the energy efficiency and economic viability of geothermal HVAC systems. Ensuring the optimal design requires accurately forecasting

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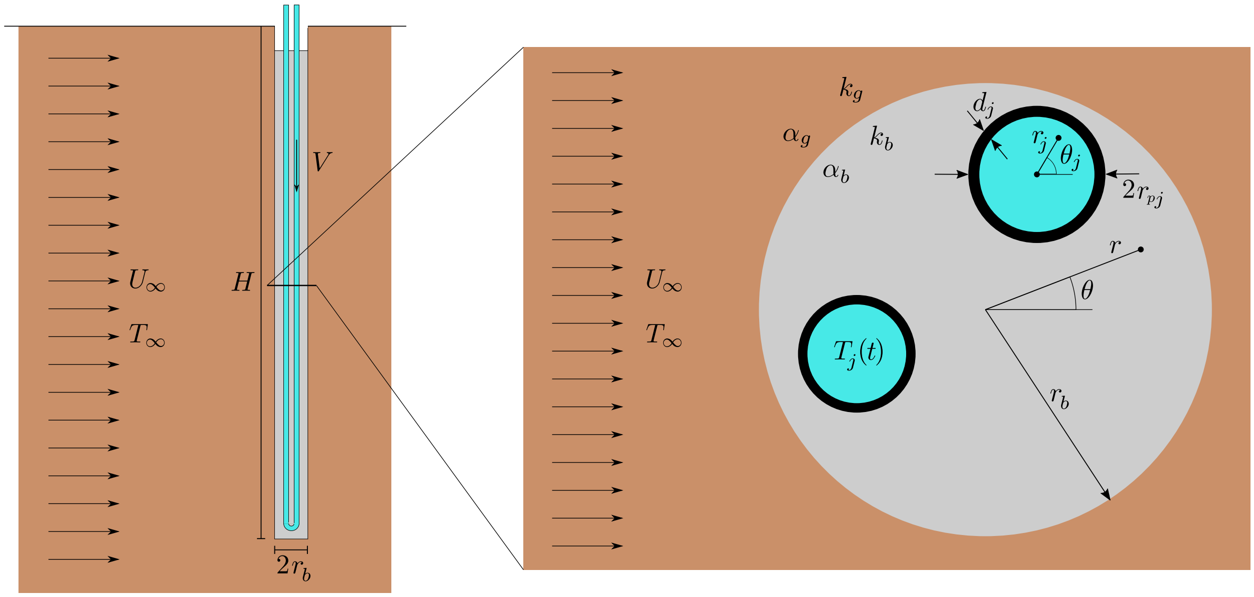

Fig. 1 Sketch of a typical geothermal borehole.

the thermal response of geothermal heat exchangers over the whole lifespan of buildings, of typically 100 years, for which simplified theoretical models are used which are accurate, flexible, and computationally affordable. These models take into account, among other things, the different heat transfer mechanisms present in the ground. In the absence of aquifers, only heat conduction acts, whereas heat convection must also be taken into account in the presence of flowing groundwaters.

Many theoretical models already exist in the literature that account for both heat transfer mechanisms [6]. They were mostly developed with creeping groundwater flows in mind for which heat convection is weak compared to heat conduction in the vicinity of the boreholes. But strong groundwater flows do also exist in real-world applications involving high-permeability soils or energy piles [3]. For such flows, current models fail to correctly account for the flow field in the vicinity of the boreholes, leading to unsatisfactory results or to the need of empirical tuning parameters [2]. To overcome these limitations, matched asymptotic expansion techniques have been used by the authors to develop a physically sound and mathematically rigorous model for the thermal interaction of geothermal boreholes with strong groundwater flows [5]. What follows is a brief overview of the developed model.

2. FORMULATION

Thanks to the slenderness of typical geothermal boreholes, the heat transfer problem in the grout filling the borehole and in the ground surrounding the borehole can be formulated in independent two-dimensional planes perpendicular to the borehole [4]. These planes, shown in Figure 1, are coupled only through the temperatures of the heat carrying liquid. Then, the grout/ground temperature T must obey the following energy conservation equations in grout and ground, respectively, in which α_b is the thermal diffusivity of the grout, α_g is the effective thermal diffusivity of the ground, (r, θ) are polar coordinates centered at the borehole, and t represents time:

$$\frac{\partial T}{\partial t} = \alpha_b \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right] \qquad \text{and} \qquad \frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} = \alpha_g \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right].$$

The effective velocity components (v_r, v_θ) of the groundwater flow are given in terms of the effective seepage velocity U_∞ of the aquifer and the borehole radius r_b by the following expressions [4]:

$$v_r = U_\infty \left(1 - \frac{r_b^2}{r^2}\right)\cos(\theta) \qquad \text{and} \qquad v_\theta = -U_\infty \left(1 + \frac{r_b^2}{r^2}\right)\sin(\theta).$$

At the borehole wall, where $r = r_b$, continuity in temperatures and normal heat fluxes is enforced through

$$T\big|_{r=r_b^-} = T\big|_{r=r_b^+} \qquad \text{and} \qquad -k_b \frac{\partial T}{\partial r}\bigg|_{r=r_b^-} = -k_g \frac{\partial T}{\partial r}\bigg|_{r=r_b^+},$$

where k_b is the thermal conductivity of the grout and k_g is the effective thermal conductivity of the ground.

Next, boundary conditions at the N_p pipes inside the borehole must be specified, for which a polar coordinate system (r_j, θ_j) centered at each pipe j is introduced. A prescribed heat injection rate per unit pipe length $q_j(t)$ is then specified at each pipe j using the following expression in which r_{pj} represents the outer radius of pipe j [4]:

$$q_j(t) = \int_{-\pi}^{\pi} -k_b \frac{\partial T}{\partial r_j}\bigg|_{r_j=r_{pj}} r_{pj}\,\mathrm{d}\theta_j.$$

This condition, however, is not mathematically sufficient, so an additional condition is enforced at each point on the outer surface of pipe j, with the bulk temperature $T_j(t)$ of the fluid in pipe j conveniently set to ensure the prescribed heat injection rate per unit pipe length $q_j(t)$ is satisfied at all times [4]:

$$-k_b r_{pj} \frac{\partial T}{\partial r_j}\bigg|_{r_j=r_{pj}} = \frac{T_j(t) - T\big|_{r_j=r_{pj}}}{R_{pj}} \qquad \text{with} \qquad T_j(t) = \frac{R_{pj}}{2\pi}\, q_j(t) + \frac{1}{2\pi}\int_{-\pi}^{\pi} T\big|_{r_j=r_{pj}}\,\mathrm{d}\theta_j.$$

The pipe's inner thermal resistance R_{pj}, appearing in the previous two expressions, accounts for heat conduction within the pipe wall and for the convective transport of heat within the fluid.

Last, the unperturbed ground temperature T_∞ is enforced far from the borehole, at $r \to \infty$, and at the beginning of the formulated heat transfer problem. That is, at $t = 0$.

3. ASYMPTOTIC SOLUTION

No exact solution is known to the formulated problem, so approximate solutions are sought instead. In the present work, matched asymptotic expansion techniques are used for that purpose. These exploit the presence of small parameters to decompose complex problems into simpler ones. For the considered problem, the small parameter is the ratio between the characteristic transversal diffusion time $t_b \sim r_b^2/\alpha_g$ and the characteristic time of variation t_q of the heating and cooling needs of the building: $t_b/t_q \ll 1$ [5]. This ratio emerges in the scale analysis of the different terms within the energy conservation equation in the ground,

$$\underbrace{\frac{\partial T}{\partial t}}_{\sim \frac{\Delta T}{t_q}} + \underbrace{v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta}}_{\sim U_\infty \frac{\Delta T}{r}} = \underbrace{\alpha_g \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right]}_{\sim \alpha_g \frac{\Delta T}{r^2}},$$

giving rise to two distinct regions. In the inner region, located at distances $r \sim r_b$, thermal inertia becomes negligible compared to heat conduction as a consequence of $t_b/t_q \ll 1$ [4, 5]. Heat convection, on the contrary, is non-negligible there due to Peclet numbers of the groundwater flow, $\mathrm{Pe} = (U_\infty r_b)/\alpha_g$, being of order unity. In the outer region, located further away from the borehole, thermal inertia recovers its importance, giving rise to a rich heat transfer problem in which the velocity field can be approximated by an uniform flow stream.

Each of the described regions is solved separately [5]. The resulting expressions, which involve Mathieu

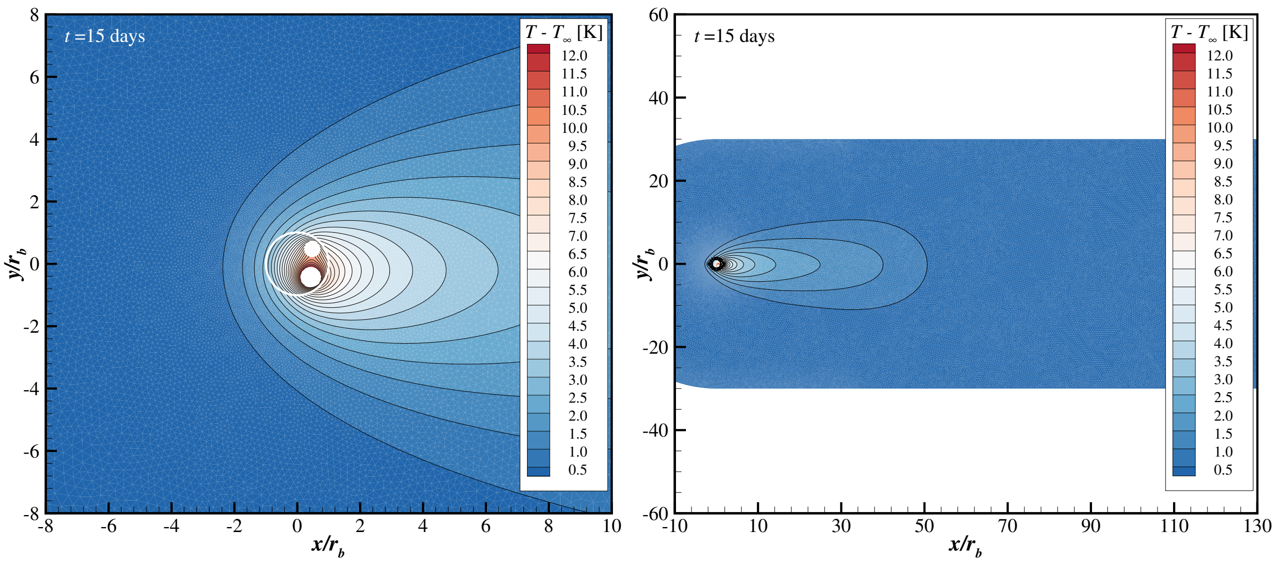

Fig. 2 (Left) inner solution and (right) outer solution for $\mathrm{Pe} = 1.0$ [5].

functions and modified Bessel functions of the second kind, are then matched at an intermediate distance between the two regions for which both solutions are valid. The final outcome is the sought model for the thermal interaction of geothermal boreholes with strong groundwater flows.

4. NUMERICAL EXAMPLE AND CONCLUSIONS

The accuracy of the developed model has been analyzed against detailed numerical simulations performed with COMSOL. Figure 2 shows in black isolines the results of the proposed model superimposed to the color map representing the results delivered by COMSOL. The chosen borehole configuration corresponds to the one analyzed by the authors in recent publications [4, 5]. The obtained inner solution, shown on the left plot, as well as the outer solution, shown on the right plot, perfectly match the results provided by COMSOL in their respective regions of validity, evidencing the capabilities of the developed model.

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